NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 160

AN AIRSHIP SLIDE RULE

By E. R. WEAVER and S. F. PICKERING

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AERONAUTICAL SYMBOLS.

1. FUNDAMENTAL AND DERIVED UNITS.

	Symbol.	Metric.		English.	
		Unit.	Symbol.	Unit.	Symbol.
Length	l	meter	m.	foot (or mile)	ft. (or mi.).
Time	t	second	sec.	second (or hour)	sec. (or hr.).
Force	F	weight of one kilogram	kg.	weight of one pound	lb.
Power	P	kg.m/sec		horsepower	HP
Speed		m/sec	m. p. s.	mi/hr	M. P. H.

2. GENERAL SYMBOLS, ETC.

Weight, $W=mg$.

Standard acceleration of gravity, $g=9.806\text{m/sec.}^2=32.172\text{ft/sec.}^2$

Mass, $m=\frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.^{3} = 0.07635 lb/ft.^{3}

Moment of inertia, mk^2 (indicate axis of the radius of gyration, k, by proper subscript).

Area, S; wing area, S_w, etc.

Gap, G

Span, b; chord length, c.

Aspect ratio $=b/c$

Distance from *c. g.* to elevator hinge, f.

Coefficient of viscosity, μ.

3. AERODYNAMICAL SYMBOLS.

True airspeed, V

Dynamic (or impact) pressure, $q=\frac{1}{2}\rho V^2$

Lift, L; absolute coefficient $C_L=\frac{L}{qS}$

Drag, D; absolute coefficient $C_D=\frac{D}{qS}$.

Cross-wind force, C; absolute coefficient

$$C_c=\frac{C}{qS}.$$

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c, D_c.)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

Dihedral angle, γ

Reynolds Number $=\rho\frac{Vl}{\mu}$, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length), C_p.

Angle of stabilizer setting with reference to lower wing. $(i_t - i_w)=\beta$

Angle of attack, α

Angle of downwash, ϵ

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Bureau of Standards

20370—23——1

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INTRODUCTION.

This report, prepared for the National Advisory Committee for Aeronautics, describes an airship slide rule developed by the Gas-Chemistry Section of the Bureau of Standards, at the request of the Bureau of Engineering of the Navy Department. The development of this slide rule was requested by the Navy because of the successful results which had been reported of the Scott-Teed rule [1] which had been developed and used by the British naval air service. It is intended primarily to give rapid solutions of a few problems of frequent occurrence in airship navigation, but it can be used to advantage in solving a great variety of problems, involving volumes, lifting powers, temperatures, pressures, altitudes, and the purity of the balloon gas.

The rule is graduated to read directly in the units actually used in making observations, constants and conversion factors being taken care of by the length and location of the scales. In order to simplify as much as possible the manipulation of the rule, absolute accuracy has in some cases been sacrificed to convenience. Generally this has been necessary only in those cases in which the data upon which the computations will be based are not subject to accurate observation.

It is thought that with this rule practically any problem likely to arise in this class of work can be readily solved after the user has become familiar with the operation of the rule; and that the solution will, in most cases, be as accurate as the data warrant.

DESCRIPTION OF RULE.

The rule, which is similar in construction to the ordinary 20-inch slide rule (fig. 1), consists of two fixed guide rails, a movable slide, and two runners with cross lines, one of which can be clamped in a fixed position. All scales are logarithmic excepting the altitude scale, which is linear.

The scale on the lower fixed guide and the lower scale on the slide (marked scales "D" and "C," respectively) read from 10 to 100. On each scale may be engraved a line V, representing the volume of the ship. Scale E is for the purity of the hydrogen and is graduated from 75 per cent to 100 per cent. Scale F is the temperature scale, reading from $-50°$ F to $+150°$ F. It represents the change of volume of a gas with change of temperature. Scale B is the altitude scale and is used to indicate the variation in lifting power at different altitudes. It reads from 0 to 25,000 feet. Scale A is the barometric scale, reading from 20 inches to 32 inches of mercury pressure. Scale K is a temperature scale, reading from $-50°$ F. to $+150°$ F., and is used to calculate the change in lifting power due to a difference between the air and gas temperatures.

[1] The Scott-Teed rule is smaller and simpler than the one described in this paper. It has a single slide and only four scales. The "A" scale gives the lift in pounds per 1,000 cubic feet of gas; the "B" scale is graduated in per cent of purity; the "C" scale in degrees Fahrenheit of temperature; and the "D" scale in inches of mercury or barometric pressure. Setting the temperature opposite the barometer one reads the lift per 1,000 cubic feet opposite the purity.

THEORY OF THE RULE.

The mathematical relations involved in the design of the rule will now be described. The lifting power of an airship is given by the equation

$$L = VP(D-d)$$

where

L represents the lifting power of the ship of volume V.

P represents the purity of the hydrogen determined from its density, assuming the impurity to be air.

D and d represent, respectively, the weight per unit volume of air and pure hydrogen.

Let T represent the temperature of the air and B the barometric pressure under which L is determined. The density of a gas varies directly as the pressure and inversely as the absolute temperature. Hence we may write for D and d

$$D = \frac{D_0 T_0 B}{T B_0} \quad \text{and} \quad d = \frac{d_0 T_0 B}{t B_0}$$

where D_0 and d_0 represent the densities of air and hydrogen at the temperature T_0 and barometric pressure B_0. We can, therefore, write for the lifting power of the airship

$$L = \frac{VPBT_0\left(\frac{D_0}{T} - \frac{d_0}{t}\right)}{B_0} \tag{1}$$

If the temperatures of gas and air are equal, this becomes

$$L = \frac{VPBT_0\ (D_0 - d_0)}{B_0 T} \tag{2}$$

Setting $\frac{T_0\ (D_0 - d_0)}{B_0} = K$, equation 2 becomes

$$L = \frac{KVPB}{T} \tag{3}$$

This equation may be written

$$\log L = \log K + \log V + \log P + \log B - \log T \tag{4}$$

In order to determine the effect of altitude upon lifting power, the values for the lifting power of 1,000 cubic feet of pure hydrogen were computed for each 1,000-foot level from the average weather data given by W. R. Gregg in the Monthly Weather Review, *46*, 11–20 (1918). The logarithms of the lifting powers so determined are plotted as abscissæ with the altitudes as ordinates. From this graph, shown in Figure 2, it is seen that the points lie approximately on a straight line, the equation of which is

$$\log L = .8537 - .01346\ H$$

where H represents the altitude above sea level. If by L we represent the lifting power at a given altitude H, and by L' the lifting power at another altitude H', then

$$\log L' = .8537 - .01346\ H'$$
$$\log L' = \log L - .01346\ (H' - H).$$

Designating $H' - H$ by h this equation becomes

$$\log L' = \log L - .01346h. \tag{5}$$

Putting this value in equation (4)

$$\log L' = \log K + \log V + \log P + \log B - \log T - .01346h \tag{6}$$

We now have all our relations in a form which permits the solution of problems involving them by means of a slide rule, which is merely a simple device for mechanically performing additions and subtractions. All of the scales representing the quantities of interest are logarithmic except that representing altitude, which is linear.

The scales are laid out as follows:

The scales C and D, upon which the lifting powers and volumes are read, are the same as those upon an ordinary 20-inch slide rule and are 50 centimeters long. Since the difference between the logarithms of 10 and 100 (the numbers at the ends of the scales) is 1, the distance in centimeters between the lines representing any two numbers on this and the other logarithmic scales is fifty times the difference between the logarithms of the numbers.

From equation 5 it is seen that the change in log L' when going to an elevation of 25,000 feet would be

$$.01346 \times 25 = .3365.$$

Multiplying this by 50 gives 16.825 centimeters as the length of scale B which is to be divided into equal intervals. The construction of six scales which will correctly represent six of the quantities involved in the solution of equation 6 has thus been determined. The scales are lettered on the slide rule and the quantities they represent in the solution of the above problem are as follows:

Scale A represents log B.
Scale B represents $.01346h$.
Scale C represents log V.
Scale D represents log L'.
Scale E represents log P.
Scale F represents log T.

The remaining factor, log K, is a constant quantity which involves the relative position of the scales to one another. Actually, in laying off the scales A, B, C, D, and E were arbitrarily placed in convenient positions and scale F was located by the accurate solution of a definite problem.

Thus far we have been concerned only with the lifting power of a definite volume of hydrogen. The airship pilot has also to deal at times with the unknown volume of a definite quantity or mass of hydrogen; that is, when the ship is not full. Scale K has been added to solve problems of this character.

In the following discussion we will let L represent the lifting power and V the volume of a given mass of hydrogen at temperature t, let d represent the density of the hydrogen, and let D represent the density of the air when the barometric pressure is B and the temperature of the air is T. We will represent by L_0, V_0, d_0, and D_0 the corresponding quantities when the temperature of hydrogen and air are both T_0 and the barometric pressure is B_0.

$$L = V\,(D-d) \tag{1}$$

From the gas laws

$$V = \frac{V_0 B_0 t}{B T_0}$$

$$d = \frac{d_0 B T_0}{B_0 t}$$

$$D = \frac{D_0 B T_0}{B_0 T}$$

$$L = V_0 \frac{B_0 t}{B T_0} \times \frac{B T_0}{B_0} \left(\frac{D_0}{T} - \frac{d_0}{t}\right)$$

$$L = V_0 t \left(\frac{D_0}{T} - \frac{d_0}{t}\right) \tag{2}$$

from which it is at once apparent that the lifting power of a given mass of hydrogen at barometric pressure is independent of the barometric pressure.

If $t=T$, equation 2 becomes

$$L=V\ (D_0-d_0). \tag{3}$$

That is, the lifting power is also independent of the temperature if the temperature of hydrogen and air are equal.

If the temperature of gas and air are not equal, equation 2 may be written

$$L=V_0\left(D_0\frac{t}{T}-d_0\right).$$

In order to determine the effect of an increment of temperature of the gas above that of the air upon the logarithm of the lifting power (which is the function with which we are concerned in designing the slide rule) we may assume T constant and write

$$\log L=\log\ V_0\left(D_0\frac{t}{T}-d_0\right). \tag{3}$$

Differentiating with respect to t

$$\frac{d\log L}{dt}=\frac{D_0}{T\left(D_0\frac{t}{T}-d_0\right)}. \tag{4}$$

Since we are never practically concerned with very great differences between gas and air temperature, t is substantially equal to T and we may write

$$\frac{d\log L}{dt}=\frac{D_0}{t(D_0-d_0)} \tag{5}$$

$$d\log L=\frac{D_0}{(D_0-d_0)}\frac{dt}{t}.$$

Integrating

$$\log L=\frac{D_0}{(D_0-d_0)}\log t+C. \tag{6}$$

Hence the relation of lifts L and L' corresponding to different gas temperatures when the ship is not full is given by the following equation

$$\log L-\log\ L'=\frac{D_0}{D_0-d_0}(\log t-\log t'). \tag{7}$$

Scale K is laid out by setting off from an arbitrary starting point the number of centimeters corresponding to the logarithm of each absolute temperature multiplied by $\frac{50\ D_0}{D_0-d_0}$. Scale K is used only in problems involving a comparison between the lifting power at two different gas temperatures. Only the distance between the lines representing the two temperatures actually enters into the solution of the problem; the location of scale K with respect to the other scales on the rule is therefore immaterial.

ERRORS INVOLVED IN THE USE OF THE RULE.

The principal theoretical errors involved in the use of the rule are as follows:

(1) The altitude scale is constructed for average weather conditions. The variation of lifting power with variation of altitude is dependent upon several factors, chief of which is the temperature. A more accurate solution of problems involving altitude could be obtained by laying out a scale or diagram in the manner indicated in Figure 3 and working with the portion of the diagram corresponding to the observed temperature. Such a scale would somewhat complicate the construction and use of the rule and would probably add but little to its utility.

The effect of altitude on volume or lifting power is probably not often desired with greater accuracy than is given by the simple scale, and complications caused by clouds, ascending or descending air currents, and other local weather conditions would render high accuracy impossible in any case.

If it should later seem desirable to include the modified scale, it may be conveniently placed on the back of the slide with a reference mark on the guide near the left-hand end where it will still serve for the solution of most practical problems.

(2) The altitude scale was laid off from the average density of the air as determined by the Weather Bureau, which differs slightly from the density computed from average temperatures and pressures, principally because humidity is taken account of in the Weather Bureau data. The altitude scale should therefore more nearly represent the true average effect of altitude on lifting power, and less nearly represent the effect of altitude on volume, than a computed scale. For this reason an altitude-volume scale was included on the rule first designed, but the difference between the two scales was so slight that it was decided to omit the altitude-volume scale. A scale was also included on the first rule to show the effect on volume of adiabatic expansion when changing altitude. Expansion is never entirely adiabatic, however, even when change of level is very rapid, so that the use of the correction for adiabatic expansion is likely to involve an assumption as far or farther from the facts than does the assumption that the gas temperature changes as rapidly as the air temperature. The adiabatic expansion-altitude scale was therefore also omitted.

(3) The effect of the average humidity of the air on lifting power is included in the construction of the altitude scale and the location of scale F. The effect of water vapor in the hydrogen can be corrected for only by regarding it as an impurity. If an electrical purity meter is employed, water vapor appears as an impurity and is very nearly correctly accounted for. If an effusion apparatus is employed for determining purity and the usual temperature corrections are made, the water vapor is not corrected for. If there is good reason to regard the hydrogen as saturated, the effusion method, uncorrected for temperature, should give more nearly the correct lifting power than it will if the temperature correction is made. One of the foreign airship slide-rules which we have had the privilege of examining provides for a correction for humidity with change of temperature. Such a correction could be easily embodied in the construction of the scales provided we knew what assumption to make regarding the change of humidity of gas and air with change of temperature. Generally, however, there is no water present to saturate the gas when the temperature rises, and water is but slowly lost to the atmosphere through the envelope when the temperature falls. It is therefore probably much more nearly correct to regard the water vapor in the hydrogen as an impurity of constant amount during any one voyage than to regard it as a variable which depends upon temperature.

(4) The correction for superheat by means of scale K is not strictly accurate because of two approximate assumptions involved in its derivation. To be exact, a different scale would have to be constructed for every air temperature. The errors introduced by this approximation are entirely negligible, however, amounting to only about 0.01 millimeter in the slide rule setting for an extreme case.

(5) A larger error is involved in the use of scale K for a gas containing more than a very small amount of impurity, since the presence of an impurity increases the value of d_0. This error is also too small to be of consequence.

(6) It is recommended that no correction be made for superheat when the ship is full of hydrogen, since the error involved when superheat is neglected is too small to be of consequence in most cases.

CHANGE IN RULE IF HELIUM IS USED.

The rule can readily be adapted for use in the case where helium instead of hydrogen is employed by multiplying by the ratio of the lifting powers of helium and hydrogen as with the ordinary slide rule. A better way, however, would be to engrave a set of scales on the reverse side of the movable slide. These scales would be identical with those used for hydrogen except-

ing that scale F would be shifted somewhat to the right. Such a rule could then be used interchangeably for hydrogen or helium without materially increasing the cost.

USE OF THE RULE.

The utility of the rule for solving problems other than the two or three special ones for which it was designed should be apparent to anyone familiar with the principles and use of slide rules. In particular the temperature scales facilitate the solution of almost any problem involving changes of gas volumes or densities. In the earlier of the following representative problems particular attention is given to illustrating the use of the temperature scales, which are the only ones likely to cause confusion. In nearly every case the volume of the airship has been assumed to be 243,000 cubic feet, corresponding to the line V marked on the slide rule illustrated. Such a line should be engraved on a rule to accompany each ship representing the volume of the ship.

Problem 1.

What will be the total lifting power L' of a ship of 243,000 cubic feet capacity at an altitude of 5,000 feet, if the barometer reading at the ground is 30 inches and the air temperature 60° F. and the hydrogen is 95 per cent pure?

(1) Opposite 30 (scale A) set 5 (scale B).
(2) Set the runner over 95 (scale E).
(3) Move the slide to bring 60 (scale F) under the runner.
(4) On scale D opposite 243 (scale C) read $L' = 14{,}040$ pounds which is the required answer.

If the lifting power of the balloon at the point of observation of temperature and barometer is desired, the barometric reading is, of course, set opposite the 0 on the altitude scale. If the lifting power of the hydrogen per thousand cubic feet is desired, it is read on scale D opposite the index of scale C, without changing the setting. (Either end of a slide rule scale reading from 1 to 10 or 10 to 100 is called the index.)

Problem 2.

How full should an airship be at the start in order to reach an altitude of 8,000 feet without losing gas?

(1) Opposite 28 (scale A) set 8 (scale B).
(2) Opposite 100 (scale C) read 78 per cent (scale D).

Problem 3.

An airship is in equilibrium at a height of 2,000 feet. The pilot estimates that the ship is 90 per cent full and that the total weight of the ship and its load is 11,000 pounds. How much ballast must be dropped to rise to a height of 6,000 feet?

(1) Opposite 11 (scale D) set 90 (scale C.)
(2) Bring runner over 100 (scale C).
(3) Move slide until 10 (scale C) is under runner.
(4) Set runner over 2,000 (scale B).
(5) Move slide until 6,000 (scale B) is under runner.
(6) Opposite 10 (scale C) read 10,790 pounds (scale D), the total lifting power of the ship at 6,000 feet.
(7) $11{,}000 - 10{,}790 = 210$ pounds, the amount of ballast which must be dropped to make the ascent.

In case the ballast to be dropped should come out a negative number it would mean that we were wrong in assuming that the balloon would be full after the ascent. If this point is in doubt, it should be solved in advance as follows:

(1) Set the index of C opposite the number in D representing in per cent the fullness of the ship.
(2) If the number (on scale B) opposite 28 (on scale A) is less than the required increase, in altitude, the ship will be full after the ascension.

Problem 4.

The total load of an airship in equilibrium is known to be 13,500 pounds when the temperature of the air is 30° F. and the temperature of the gas 45° F. If no gas is lost until after sundown, when the temperature of gas and air will become equal, how much will the total lifting power of the balloon then be?

(1) Opposite 135 (on scale D) set the index of scale C.
(2) Bring the runner over 30 (scale K).
(3) Move the slide until 45 (scale K) comes under the runner.
(4) Opposite the index of C read 12,770 pounds (on scale D), the lifting power of the balloon after sunset.

The second runner, which can be clamped in a fixed position on the rule, is provided for use in connection with such problems as this and the one following. If the lifting power of the ship is determined under any known conditions, a setting of the slide may be made corresponding to the operations in problem 1. This position of the index of scale C then represents the lift if the air and gas temperatures become equal. If the auxiliary runner is clamped over this index, the lifting power of the ship under any subsequent conditions of superheat of the gas may be determined by bringing the index of the slide under the auxiliary runner again and performing the operations corresponding to 2, 3, and 4 in problem 4.

Problem 5.

When an airship of 243,000 cubic feet capacity reaches the summit of its flight, the barometer is observed to read 22 inches, the temperature of the gas is 30° F. and its purity 98 per cent. What will be the lifting power of the ship when the air temperature is 50° and the gas temperature 65°?

(1) Opposite 22 (scale A) set 0 (scale B).
(2) Set the runner over 98 (scale E).
(3) Move the slide to bring 30 (scale F) under the runner.
(4) Set the runner over 65 (scale K).
(5) Move the slide to bring 50 (scale K) under the runner.
(6) Opposite 243 (scale C) read 13,630 (scale D), which is the required lifting power.

Problem 6.

How high will the airship of 243,000 cubic feet capacity rise with a load of 9,000 pounds if it is filled with 98 per cent hydrogen, the barometer reads 25 inches, and the air temperature is 80° F.?

Solution:

(1) Opposite 9,000 (scale D) set V (243 on scale C).
(2) Set the runner over 80 (scale F).
(3) Shift the slide to bring 98 under the runner.
(4) Set the runner over the index of scale C.
(5) Shift the slide to bring the other index of scale C under the runner.
(6) Opposite 25 (scale A) read 13,200 (scale B), which is the altitude to which the ship will rise.

Problem 7.

The total weight of the ship of 243,000 cubic feet capacity and its load is 15,000 pounds. It is just in equilibrium at a barometric pressure of 31 inches and an air temperature of 50° F. The purity of the hydrogen is 94 per cent. How much will the lifting power of the ship be increased if pure hydrogen is added until the bag is full?

Solution:

(1) Opposite 31 (scale A) set 0 (scale B).
(2) Set runner R over 94 (scale E).
(3) Shift slide to bring 50 (scale F) under R.

Solution—Continued.

(4) Set runner R over index of scale C. This gives (on scale D) the lifting power per thousand cubic feet of the 94 per cent hydrogen.

(5) Shift slide until 150 is under the runner.

(6) On scale C opposite the index of scale D read 213,000, the number of cubic feet of gas in the bag.

(7) 243,000 − 213,000 = 30,000, which is the number of cubic feet of pure hydrogen added.

(8) Shift the slide to bring the index of C under the runner.

(9) Set the runner over 100 (scale E).

(10) Shift the slide to bring 94 (scale E) under the runner.

(11) Opposite 30,000 (on scale C) read 2,245 (on scale B), which is the amount by which the lifting power of the ship has been increased.

Problem 8.

A balloon in the hangar is to be filled to rise to a total altitude of 5,000 feet in bright sunshine. The observed temperature of the air is 70°; assume that it is known from experience that bright sunlight heats the gas to a temperature of 20° F. above that of the surrounding air. How much hydrogen should the balloon contain in order that it will be full at the desired altitude?

Solution:

(1) Opposite 28 (scale A) set 5,000 (scale B).

(2) Set the runner over 70 (scale F).

(3) Shift the slide to bring 90 (scale F) under the runner.

(4) Opposite V (243 on scale C) read 201,000 cubic feet (on scale D), which is the volume of hydrogen required.

Problem 9.

An airship of 243,000 cubic feet capacity is to be filled from cylinders into which the hydrogen was compressed at a pressure of 1,600 pounds per square inch and a temperature of 90° F. If the cylinders are known to deliver, when filled to a pressure of 1,800 pounds, exactly 100 cubic feet of gas measured at 30 inches barometric pressure and 63°F., and the observed barometric pressure is 27 inches and the observed temperature 30° F. at the time of use, how many cylinders must be used to fill the ship?

Solution:

(1) Opposite 18 (scale D) set 16 (scale C).

(2) Set the runner over 90 (scale F).

(3) Shift the slide to bring 30 (scale F) under the runner.

(4) Set the runner over 27 (scale C).

(5) Shift the slide to bring 30 (scale C) under the runner.

(6) Opposite 243 (scale C) read 2,457, the number of cylinders required.

The above problem illustrates the utility of the rule in solving all problems involving the change of the volume of gas with change of temperature. The temperature scale F can be used for any such computations without the necessity of reducing observed temperatures to the absolute scale.

Problem 10.

An observation balloon of 30,000 cubic foot capacity is to be sent to a height of 9,000 feet. It is to be filled from a generator producing a maximum of 10,000 cubic feet of gas per hour. How much time is wasted if the balloon is completely filled before ascent?

Solution:

(1) Opposite 28, set 9,000.

(2) Opposite 180 on scale C (the time in minutes required to fill the balloon, computed mentally) read 136 minutes (scale D), which is the time in minutes required to generate enough gas to fill the balloon at 9,000 feet.

(3) 180 − 136 = 44 minutes, the time lost in filling the balloon to capacity before the ascent.

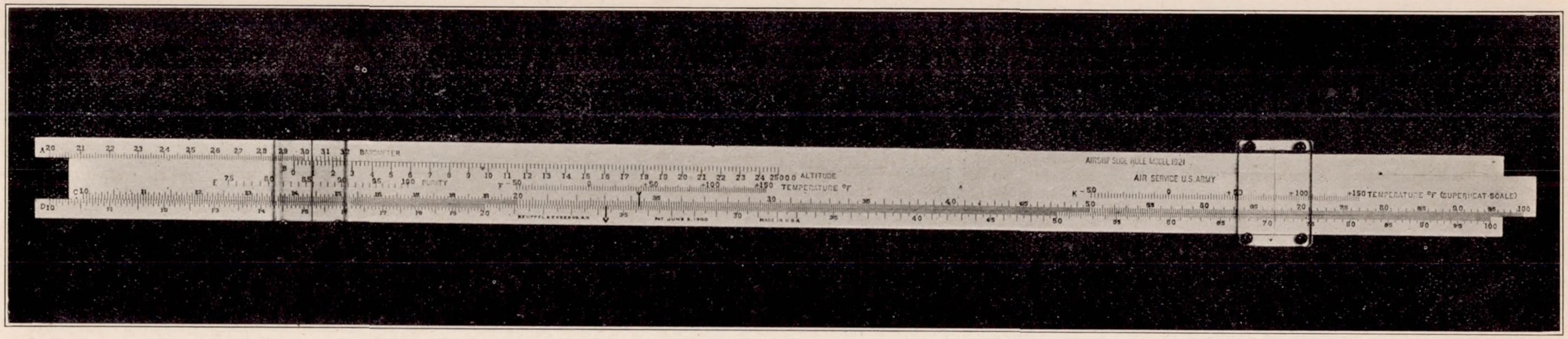

FIGURE 1.

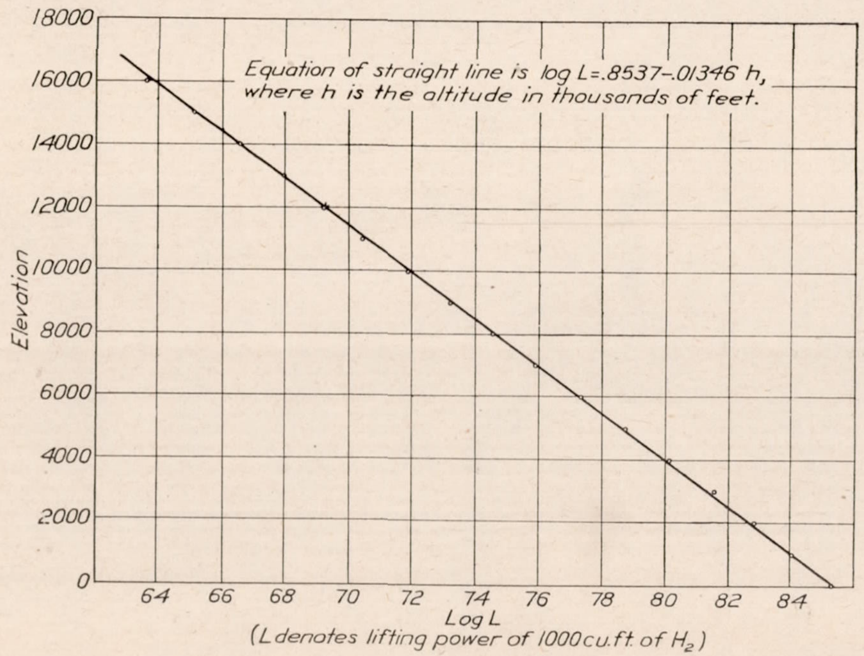

FIG. 2.

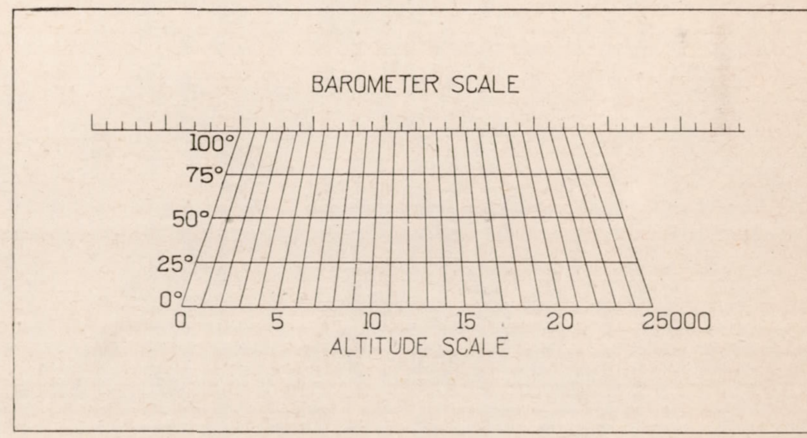

FIG 3

Problem 11.

An airship carrying a total weight of 12,000 pounds in sunlight is to make a landing at night. The temperature of the air is 40° and that of the gas 60°. The gas bag is 95 per cent full. The ship has only 800 pounds of ballast. How much higher can the ship rise and still retain enough ballast to enable it to remain afloat after dark?

Solution:

(1) 12,000 − 800 = 11,200.
(2) Opposite 11,200 (scale D) set 12,000 (scale C).
(3) Set the runner over 95 (scale C).
(4) Shift the slide to bring the index of scale C under the runner.
(5) Set the runner over 60 (scale K).
(6) Shift the slide to bring 40 (scale K) under the runner.
(7) Opposite 28 (scale A) read 2,500 feet, the permissible ascent (scale B).

CONCLUSION.

Other problems in great variety, not included in the foregoing set, have been shown to be capable of solution with this rule, and the authors feel that it is not only applicable to a greater range of problems but is much simpler in its operation than any of the foreign makes which have been examined.

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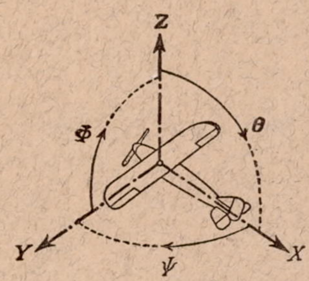

Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Symbol.		Designation.	Symbol.	Positive direction.	Designation.	Symbol.	Linear (component along axis).	Angular.
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	Φ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	Θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	Ψ	w	r

Absolute coefficients of moment

$$C_l=\frac{L}{q\,b\,S} \quad C_m=\frac{M}{q\,c\,S} \quad C_n=\frac{N}{q\,f\,S}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D
Pitch (a) Aerodynamic pitch, p_a
(b) Effective pitch, p_e
(c) Mean geometric pitch, p_g
(d) Virtual pitch, p_v
(e) Standard pitch, p_s
Pitch ratio, p/D
Inflow velocity, V'
Slipstream velocity, V_s

Thrust, T
Torque, Q
Power, P
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T\,V/P$
Revolutions per sec., n; per min., N
Effective helix angle $\Phi = \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.44704 m/sec.
1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.
1 kg. = 2.20462 lb.
1 mi. = 1609.35 m. = 5280 ft.
1 m. = 3.28083 ft.